# A New Look at Organ Transplantation Models and Double Matching Queues

Hybrid of queueing and inventory aspects.

**Features**:

* Servers are organs 🡪 sequential and random arrivals,
* The transplantation only takes a negligible time.
  + Double matching queue 🡪 2 lines (organs and patients).
* Both queues are affected by impatience 🡪 reductions of lengths without matchings taking place:
  + health deterioration of waiting patients 🡪 random positive time-lag, drawn from a general distribution assigned to any arriving customer (patient).
  + live organs cannot be preserved for more than a certain fixed period of time.
    - stochastic inventory theory 🡪 **perishable inventory system** (PIS) whose output process is split into satisfied demands and outdatings.

**Model**

* **Organs arrive** at the organ bank 🡪 renewal process with inter-renewal time distribution function (mean and Laplace-Stieltjes transform (LST) ).
* **Patients arrivals** 🡪 Poisson process of rate , independent of the item arrival times.
* **Allocation**:
  + Demands that arrive at a **not-empty organ bank** are satisfied immediately by the oldest organs.
  + Demands that arrive at an **empty organ bank** join the line of waiting demands.
  + Newly arriving organs are assigned on the spot to waiting demands on a FCFS basis.
* **Patience time** 🡪 time until the patient's physical condition no longer allows carrying out a transplantation.
  + 🡪 patience time of the n-th arriving patient 🡪 distribution function (mean and LST )
  + 🡪 iid, positive random variables, independent of the arrival times of organs and demands.
* **Shelf lifetime** of the stored organs 🡪 prespecified constant equal to 1.
* 2 connected queueing systems of the FCFS type.
* 5 performance criteria are key characteristics of the **efficiency of the organ bank**:

1. rate of the times of **organ outdatings**;

2. rate of the times at which **demands leave unsatisfied**;

3. steady-state law of the **number of organs on the shelf**;

4. steady-state law of the **waiting time of a patient**;

5. length and long-run fraction of time during which the **shelf is empty of organs**.

**Notation**:

* is the arrival time of the nth organ;
* is the arrival time of the nth demand;
* is the patience time of the nth patient;
* is the time of the nth outdating.

🡪 age process

* (shelf is not empty) shelf age of the oldest organ in the system;
* (shelf is empty) negative of the waiting time of a (virtual) patient arriving at time whose patience would be long enough to eventually receive the first available organ.

Immagine che contiene diagramma

Descrizione generata automaticamente

***Virtual outdating process*** (VOP) 🡪 🡪 time from till the next outdating of an organ that would occur if the demand arrival process was completely stopped at .

* Demands arrive according to a Poisson process with rate 🡪 is a Markov process whose sample path can also be interpreted as the workload process of a special type queueing system with impatience.

Immagine che contiene diagramma

Descrizione generata automaticamente

* 🡪 waiting time of the n-th arriving customer with patience .
  + 🡪 the n-th customer is admitted to the system
  + 🡪 the n-th customer is refused if and admitted otherwise
* queue.
  + The idle periods are deleted and the busy periods are glued together 🡪 the workload process has the same law as .

Given , the age of the oldest organ on the shelf at time t is given by .

* Outdating times 🡪 renewal process whose inter-renewal times have the same law as that of a busy period in that queue.
* Unsatisfied demand process (counts the abandoned customers) 🡪 not a renewal process.
* From the law of the outdating process derive 🡪
  + The rate of unsatisfied demands is obtained from the **conservation law** 🡪 .
    - 🡪 arrival rate of organs into the system 🡪 departure rate out of the system in steady state;
    - 🡪 the long-run average rate of organs which are not outdated (used to satisfy demands),
    - 🡪 the rate of satisfied demands.
* .

**Extensions**

* Politiche di allocazione in base alla compatibilità del gruppo sanguigno 🡪 il criterio basato sul tempo d’attesa per dare priorità ai pazienti è ancora quello dominante.
* Politiche di allocazione in base alle condizioni del paziente 🡪 priorità per stato di salute
* Tasso di arrivo degli organi variabile
* Tempo di vita degli organi non costante 🡪 ad intervalli costanti si controlla lo stato degli organi e questi vengono rimossi con probabilità *p* per deperimento.